Descomposición elemental (ajustes por constantes)

OBSERVACIONES

1. Las primeras integrales que aparecen se han obtenido del libro de Matemáticas I (1º de Bachillerato) McGraw-Hill, Madrid 2007.
2. Otros problemas se han obtenido de las Pruebas de Selectividad.

Algunas integrales con solución.

1. $\int 4x^2 dx = \frac{4}{3}x^3 + c$

2. $\int 2x^2 dx = \frac{2x^3}{3} + c$

3. $2\int (x^2 - 1)dx = \frac{2x^3}{3} - 2x + c$

4. $\int (-4)dx = -4x + c$

5. $\int (4x^2 - 3x + 4)dx = \frac{4}{3}x^3 - \frac{3}{2}x^2 + 4x + c$

6. $\int (2x^3 - 5)dx = \frac{x^4}{2} - 5x + c$

7. $\int (-3x^2 + x - 1)dx = -x^3 + \frac{x^2}{2} - x + c$

8. $\int \frac{3x+4}{5}dx = \frac{3}{10}x^2 + \frac{4}{5}x + c$

9. $\int \frac{x^2 - 2x + 1}{3}dx = \frac{x^3}{9} - \frac{1}{3}x^2 + \frac{1}{3}x + c$

10. $\frac{1}{2}\int (x^2 + 2x - 1)dx = \frac{1}{6}x^3 + \frac{1}{2}x^2 - \frac{1}{2}x + c$

11. $\int 3x(4x^2 - 3x + 4)dx = 3x^4 - 3x^3 + 6x^2 + c$

12. $\int x^2(3x - 5)dx = \frac{3}{4}x^4 - \frac{5}{3}x^3 + c$

13. $\int x(3x - 5)^2 dx = \frac{9}{4}x^4 - 10x^3 + \frac{25}{2}x^2 + c$

14. $\int \frac{-1}{x^2}dx = \frac{1}{x} + c$

15. $\int \frac{2}{x^3}dx = -\frac{1}{x^2} + c$

16. $\int \frac{-3}{x^4}dx = \frac{1}{x^3} + c$

17. $\int \frac{4}{x^5}dx = -\frac{1}{x^4} + c$

18. $\int x\sqrt{x}dx = \frac{2}{5}x^{5/2} + c$

19. $\int \frac{\sqrt{x}}{x}dx = 2x^{1/2} + c$

20. $\int \frac{x}{\sqrt{x}}dx = \frac{2}{3}x^{3/2} + c$

21. $\int \frac{3}{x}dx = 3\ln x + c$

22. $\int \frac{3}{x-1}dx = 3\ln(x-1) + c$

23. $\int \frac{1}{2x-1}dx = \frac{1}{2}\ln(2x-1) + c$

24. $\int \frac{3}{2x-1}dx = \frac{3}{2}\ln(2x-1) + c$

25. $\int \frac{3x+4}{x}dx = 3x + 4\ln x + c$

26. $\int \frac{x^2 - 2x + 1}{3x}dx = \frac{x^2}{6} - \frac{2}{3}x + \frac{1}{3}\ln x + c$

27. $\int (3+x)^4 dx = \frac{(3+x)^5}{5} + c$

28. $\int (2x^3 - 1)^5 \cdot 6x^2 dx = \frac{(2x^3 - 1)^6}{6} + c$

29. $\int \frac{2x}{x^2 + 6}dx = \ln(x^2 + 6) + c$

30. $\int \frac{1}{2\sqrt{x}}\cos\sqrt{x}dx = sen\sqrt{x} + c$

31. $18\int \frac{1}{3x+1}dx = 6\ln(3x+1) + c$

32. $\int \frac{x}{x^2 + 6}dx = \frac{1}{2}\ln(x^2 + 6) + c$

33. $\int \frac{2x-3}{x^2 - 3x}dx = \ln(x^2 - 3x) + c$

34. $\int 6e^x dx = 6e^x + c$

35. $\int 6e^{3x}dx = 2e^{3x} + c$

36. $\int 4e^{3x}dx = \frac{4}{3}e^{3x} + c$

37. $\int 4e^{2x+3}dx = 2e^{2x+3} + c$

38. $\int (2e^x - 1)dx = 2e^x - x + c$

39. $\int (2e^{2x} + x)dx = e^{2x} + \frac{x^2}{2} + c$

40. $\int 2(e^{2x} + x)dx = e^{2x} + x^2 + c$

41. $\int \frac{2e^{2x} + x}{3}dx = \frac{1}{3}e^{2x} + \frac{1}{6}x^2 + c$

42. $\int 2\cos x dx = 2senx + c$

43. $\int \cos 2x dx = \frac{1}{2}sen2x + c$

44. $\int (-5\cos 3x)dx = -\frac{5}{3}sen3x + c$

45. $\int \frac{\cos 4x}{3}dx = \frac{1}{12}sen4x + c$

46. $\int 3sen3xdx = -\cos 3x + c$

47. $\int 2sen4xdx = -\frac{1}{2}\cos 4x + c$

48. $\int (-2sen5x)dx = \frac{2}{5}\cos 5x + c$

49. $\int sen\frac{3}{2}xdx = -\frac{2}{3}\cos\frac{3}{2}x + c$

50. $\int (2 + 2tag^2 x)dx = 2tagx + c$

53. $\int \frac{-1}{\cos^2 x}dx = -tagx + c$

54 $\int tagxdx = -\ln\cos x + c$

Integrales resueltas

Integrales inmediatas

1. Calcula las siguientes integrales:

a) $\int \left(3x^2 + x - 2\sqrt{x}\right)dx$ b) $\int x\left(4 - 4x^2\right)dx$ c) $\int \frac{e^{-2x}}{5}dx$

d) $\int \frac{5x}{3+3x^2}dx$ e) $\int \cos(4x+3)dx$ f) $\int \left(\sin 2x - \frac{1}{3}\cos 5x\right)dx$

g) $\int \left(3\cos\frac{x}{2} - \frac{\text{sen } 2x}{5}\right)dx$ h) $\int x\cos\left(3x^2\right)dx$ i) $\int \left(\cos(2x) - 3e^{2x-3}\right)dx$

j) $\int \cos x\cdot(\sin x)^2 dx$ k) $\int 5x\left(1-2x^2\right)^2 dx$ l) $\int (2-3x)^2 dx$

m) $\int \frac{x^2}{x^3+2}dx$ n) $\int \frac{3}{1+x^2}dx$ o) $\int \frac{4x^2}{\sqrt{3-x^3}}dx$

p) $\int \frac{5x}{\sqrt{1-x^2}}dx$ q) $\int \frac{5}{\sqrt{1-x^2}}dx$ r) $\int 2xe^{3x^2}dx$

s) $\int (1-x)^3 dx$ t) $\int x(1-x)^3 dx$ u) $\int \frac{(x-1)^3}{x}dx$

Solución:

En la mayoría de los casos hay que ajustar constantes y operar cuando sea necesario.

a) $\int \left(3x^2 + x - 2\sqrt{x}\right)dx = x^3 + \frac{1}{2}x^2 - 2\int x^{1/2}dx = x^3 + \frac{1}{2}x^2 - 2\frac{x^{3/2}}{3/2} + c$

b) $\int x\left(4-4x^2\right)dx = \int \left(4x - 4x^3\right)dx = 2x^2 - x^4 + c$

c) $\int \frac{e^{-2x}}{5}dx = \frac{1}{5}\cdot\frac{1}{(-2)}\int(-2)e^{-2x}dx = -\frac{1}{10}e^{-2x}+c$

d) $\int \frac{5x}{3+3x^2}dx = \frac{5}{6}\int\frac{6x}{3+3x^2}dx = \frac{5}{6}\ln\left(3+3x^2\right)+c$

e) $\int \cos(4x+3)dx = \frac{1}{4}\int 4\cos(4x+3)dx = \frac{1}{4}\sin(4x+3)+c$

f) $\int\left(\sin 2x-\frac{1}{3}\cos 5x\right)dx = \frac{1}{2}\int 2\sin 2xdx-\frac{1}{3}\cdot\frac{1}{5}\int 5\cos 5xdx = -\frac{1}{2}\cos 2x-\frac{1}{15}\sin 5x+c$

g) $\int\left(3\cos\frac{x}{2}-\frac{\text{sen } 2x}{5}\right)dx = 6\int\left(\frac{1}{2}\cos\frac{x}{2}\right)dx-\frac{1}{10}\int 2\sin 2xdx = 6\sin\frac{x}{2}+\frac{1}{10}\cos 2x+c$

h) $\int x\cos\left(3x^2\right)dx = \frac{1}{6}\int 6x\cos\left(3x^2\right)dx = \frac{1}{6}\sin\left(3x^2\right)+c$

i) Ajustando constantes en cada una de las funciones:

$$\int\left(\cos(2x)-3e^{2x-3}\right)dx = \int\cos(2x)dx-\int\left(3e^{2x-3}\right)dx = \frac{1}{2}\int 2\cos(2x)dx-\frac{3}{2}\int\left(2e^{2x-3}\right)dx =$$

$$= \frac{1}{2}\sin(2x)-\frac{3}{2}e^{2x-3}+c$$

j) Ajustando constantes:

$$\int\cos x\cdot(\sin x)^2dx = \frac{1}{3}\int 3(\sin x)^2\cdot\cos xdx = \frac{1}{3}(\sin x)^3+c$$

k) $\int 5x\left(1-2x^2\right)^2dx = -\frac{5}{4}\int\left(-4x\left(1-2x^2\right)^2\right)dx = -\frac{5}{4}\cdot\frac{\left(1-2x^2\right)^3}{3}+c = -\frac{5}{12}\left(1-2x^2\right)^3+c$

l) Se opera en el integrando:

$$\int(2-3x)^2dx = \int\left(4-12x+9x^2\right)dx = 4x-6x^2+3x^3+c$$

m) Ajustando constantes:

$$\int\frac{x^2}{x^3+2}dx = \frac{1}{3}\int\frac{3x^2}{x^3+2}dx = \frac{1}{3}\ln\left(x^3+2\right)+c$$

n) Ajustando constantes:

$$\int\frac{4x^2}{\sqrt{3-x^3}}dx = -\frac{4\cdot 2}{3}\int\frac{-3x^2}{2\sqrt{3-x^3}}dx = -\frac{8}{3}\int\frac{-3x^2}{2\sqrt{3-x^3}}dx = -\frac{8}{3}\sqrt{3-x^3}+c\,.$$

o) Ajustando constantes:

$$\int\frac{5x}{\sqrt{1-x^2}}dx = -5\int\frac{-2x}{2\sqrt{1-x^2}}dx = -5\sqrt{1-x^2}+c$$

p) $\int \frac{3}{1+x^2}dx$. Es inmediata: $\int \frac{3}{1+x^2}dx = 3\cdot\int \frac{1}{1+x^2}dx = 3\arctan x + c$

q) $\int \frac{5}{\sqrt{1-x^2}}dx$. Es inmediata: $\int \frac{5}{\sqrt{1-x^2}}dx = 5\int \frac{1}{\sqrt{1-x^2}}dx = 5\arcsin x + c$

r) Ajustando constantes:

$$\int 2xe^{3x^2}dx = \frac{2}{6}\int 6xe^{3x^2}dx = \frac{1}{3}e^{3x^2} + c$$

s) Desarrollando el integrando:

$$\int (1-x)^3 dx = \int \left(1-3x+3x^2-x^3\right)dx = x - \frac{3}{2}x^2 + x^3 - \frac{1}{4}x^4 + c$$

También podría hacerse directamente ajustando constantes:

$$\int (1-x)^3 dx = -\int (-1)(1-x)^3 dx = -\frac{(1-x)^4}{4} + c$$

t) Hay que desarrollar el cubo, multiplicar e integrar: $\int x(1-x)^3 dx =$

$$= \int x\left(1-3x+3x^2-x^3\right)dx = \int \left(x-3x^2+3x^3-x^4\right)dx = \frac{1}{2}x^2 - x^3 + \frac{3}{4}x^4 - \frac{1}{5}x^5 + c$$

u) Hay que desarrollar el cubo, dividir e integrar:

$$\int \frac{(x-1)^3}{x}dx = \int \frac{1-3x+3x^2-x^3}{x}dx = \int \left(\frac{1}{x} - 3 + 3x - x^2\right)dx = \ln x - 3x + \frac{3}{2}x^2 - \frac{1}{3}x^3 + c$$